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Fundamentals of the firm and Beta estimation

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ABSTRACT

For publicly traded firms, it is common to estimate the beta and the cost of equity assuming the perspective of the marginal investor. Unfortunately, the same assumption cannot be applied directly to the private firms. Indeed, it is quite common that investor is the entrepreneur and he/she is not diversified. Also, for the private firms there are not a historical price information that are needed to estimate beta. One way to solve the problem is the "fundamental beta". It estimates the Beta of the firm on the basis of its fundamentals by applying the coefficient estimates on the basis of the analysis developed on the on the publicly traded firms. In this paper we estimate the beta on the fundamentals of the U.S. publicly traded firms in the year 2022.

Per le società quotate in borsa, è comune stimare il beta e il costo del capitale proprio assumendo la prospettiva dell'investitore marginale. Sfortunatamente, lo stesso presupposto non può essere applicato direttamente alle imprese private. In effetti, è abbastanza comune che l'investitore sia l'imprenditore e non sia diversificato. Inoltre, per le aziende private non ci sono informazioni storiche sui prezzi che sono necessarie per stimare il Beta. Un modo per risolvere il problema è il "beta fondamentale". Stima il Beta dell'impresa considerando i suoi fondamentali applicando le stime dei coefficienti sulla base dell'analisi sviluppata sulle società quotate in borsa. In questo documento stimiamo il beta sui fondamentali delle aziende statunitensi quotate in borsa nell'anno 2022.

Keywords: fundamental beta, cost of equity, private firms, valuation, cost of capital.

1 – Introduction

For publicly traded firms, it is common to estimate the cost of equity assuming the perspective of the marginal investor. It implies that the investors are well-diversified and then they compute the risk in terms of risk adds on to a diversified portfolio or market risk. It allows us to use the theory of Capital Asset Pricing Model (Sharpe (1964), Lintner (1965), Mossin (1966)).

Unfortunately, the same reasoning cannot be applied directly to the private firms for two main reasons: first, it is quite common that investor is the entrepreneur and he/she is not diversified; second, there are not a historical price information that are need to estimate beta by applying the running regression of stock returns against market return, leaving aside all problems short-term fluctuations and market anomalies (Roll, 1977).

One way to solve the problem is to estimate the beta on the basis of the fundamentals of the firm. This approach, called fundamental beta, is based on the economic and financial fundamental variables of the company, its market and macroeconomic factors (Kaplan & Ruback, 1995). The use of the fundamental beta can be applied for private firm and tend to be less affected by the possible short-term market fluctuation (Liu & Zhang, 2008). The fundamental beta estimates the beta of the private firm on the basis of the relationship between the betas of the publicly traded firms and their fundamental variables.

Because both betas and fundamental variables of the publicly traded firms are observable, if we find a statistically significant relation between beta (dependent variables) and the fundamental variables of the firm (independent variables), we can apply this relation to the private firm. Therefore, for each private firm we can estimate its beta by applying the found relationship and computing the fundamental variables of the specific firm analyzed. Among others, the study of Beaver, Kettler, and Scholes (1970) examines the relationship between betas and seven variables: dividend payout, asset growth, leverage, liquidity, asset size, earnings variability, and the accounting beta. Rosenberg and Guy (1976) also attempted a similar analysis.

Applying this regression to the data of U.S. companies in January 2011, Damodaran (2012), finds a relation between beta and some fundamental variables of publicly traded firms: ROE (-0.04), the ratio of Fixed Assets on Total Assets (+0.167), the ratio of the Book Value of Debt on the sum of the Book value of equity and debt (+0,17), the expected annual growth rate in net profit over next five years (+0,74) and effective tax rate (-0,31), with an initial intercept value equal to +0.93. The main problem of this type of regressions is the low value of the R-squared of the regression that in the case of Damodaran is equal to 9.3%.

In this paper we use the data related to the U.S. publicly traded firms with regard the year 2022 to find a significant relationship between the beta and the fundamental variables of the firm. The analysis developed find a significant relationship between beta and seven fundamental variables of the firm that are: ROE, ROA, Book Value of Debt, COGS on Total Costs, Fixed Asset on Total Asset, and change in Earnings.

Several studies have attempted to estimate firm beta using fundamental variables, obtaining variable results in terms of statistical significance. Damodaran (2012), for example, found a relationship between beta and some fundamental variables, such as ROE and the ratio of fixed assets to total assets, but with an R-square of only 0.093, signaling limited explanatory power. Similarly, Beaver, Kettler, and Scholes (1970) explored seven fundamental variables, including leverage and asset growth, finding a significant relationship with some of these variables, but with not entirely consistent results for all of them. Rosenberg and Guy (1976) also attempted to predict beta using fundamentals, but with a weak correlation between the variables and beta.

In our study, however, the R-square of the regression is 0.31, a significantly higher value than in previous studies. This suggests that our model, which includes variables such as the ratio of cost of goods sold (COGS) to total costs, and the change in earnings (Δ Earnings), is better able to explain the variability of beta than traditional models.

The structure of this paper is organised as follows:

– Section 2) – Literature Review provides an overview of the existing literature on Beta, discussing the theoretical foundations and main results of previous studies.

– Section 3) Methodology describes the process of computation the Fundamental Beta, including the selection of fundamental variables of the firm, data sources and statistical technique used and the results of the empirical analysis on fundamental beta are presented, analysing the stability and accuracy of this measure about firm fundamentals, interpreting the results obtained.

– Section 4) Conclusions summarise the main findings of the study and suggest the direction for future research and practical applications of fundamental beta by exploring the practical implications for investors and portfolio managers.

2 – Literature Review

The beta based on the Capital Asset Pricing Model (CAPM) (Sharpe (1964), Lintner (1965), Mossin (1966)) measures the systematic risk of a security relative to the market. It represents the sensitivity of a stock's return to market movements, building on the earlier work of Markowitz (1952) on portfolio theory.

Malkiel (1995) and Fama & French (2004) critically examined the beta, highlighting its variability over time and limited predictive power during financial turbulence. Blume (1971) demonstrated that estimated betas tend to regress toward the market average over time, indicating a lack of stability. Scholes and Williams (1977) identified significant estimation problems caused by non-synchronous market data, which can lead to inaccurate beta measurements. More recently, Bali and Cakici (2008) emphasized that beta may not adequately capture idiosyncratic risk, leading to an underestimation of a stock's overall risk.

In this contest the main problem is that the beta can be directly estimated in the case of private firms. The use of the fundamental beta can be a possible solution. Also, the fundamental beta seems to be able to reduce the effects of the short-run market volatility because the fundamental variables of the firms tend to be less affected by those market's volatility in short-run, and it has the relevant advantage to be used to compute the beta for the private firms.

Gebhardt, Lee, and Swaminathan (2001) were among the pioneers in this field, using firm variables to estimate credit risk. Ang et al. (2006) further demonstrated that incorporating fundamentals can improve stock return forecasts.

Several empirical studies have validated the effectiveness of Fundamental Beta. Barbee, Mukherji, and Raines (1996) found that using fundamental variables, such as financial leverage and price-to-earnings ratio, can enhance risk estimation. More recently, Hsu, Kalesnik, and Kose (2017) showed that a Beta based on fundamental variables provides more stable and accurate forecasts than a market Beta, especially during periods of high volatility.

The fundamental beta is based on the idea that the fundamentals of a company (such as operating and net profitability, assets, size, debt, etc.) can use to provide an accurate view of systematic risk. Gebhardt, Lee, and Swaminathan (2001) used firm's fundamental variables to estimate credit risk, demonstrating that firm's economic and financial information can improve risk forecasts beyond market data alone. Ang et al. (2006) further supported this idea by showing that fundamentals can enhance stock return predictions, suggesting that basic economic information is crucial for an accurate assessment of risk.

3 – Data and Methodologies

Researchers have attempted to associate the betas of publicly traded companies with observable variables such as the debt ratio and the variance of earnings. Beaver et al. (1970) analyzed the relationship between Betas and seven variables: dividend distribution, asset growth, financial leverage, liquidity, asset size, earnings variability, and accounting Beta. A similar analysis was carried out by Rosenberg and Guy (1976).

In this paper, we test our hypotheses on a sample of publicly traded American companies in 2022. The main objective is to determine how factors such as ROE, ROA, BVofdebt, COGS/Total Costs, Fixed Asset/Total Asset, and ΔEarnings influence the variability of Beta. The corporate data for the independent variables are sourced from the Orbis database, while the data for the dependent variable Beta are sourced from the S&P500 stock index. Before proceeding with the analysis, the data were cleaned by eliminating observations with missing values. Our final data sample comprises 95 observations.

To achieve our goal, based on the models proposed by Beaver et al. (1970) and Glova (2014), we developed our multiple linear regression model in which we analysed the relationship between Beta and six variables, also assessing the reliability of our regression model through the calculation of multicollinearity, autocorrelation, and heteroscedasticity. Our regression model is as follows:

$$\beta = \beta_0 + \beta_1 ROE + \beta_2 ROA + \beta_3 L + \beta_4 \frac{COGS}{TC} + \beta_5 \frac{FA}{TA} + \beta_6 \Delta Er + \varepsilon_i \qquad [1]$$

where:

- ROE, is the Return on Equity;
- ROA, is the Return on Assets;
- L, is the book value of the leverage equal to the financial debt on the capital structure (equity plus financial debt);
- $\frac{COGS}{TC}$, is the ratio between Cost of Goods Sold (COGS) and the Total Costs of the firm;
- $\frac{FA}{TA}$, is the ratio between Fixed Assets and the Total Assets of the firm;
- ΔEr, is the change in Earnings from the year 2021 to year 2022.

Our dependent variable is represented by the Beta equal to covariance between the i-th asset and the market returns (σ_{iM}) on the variance of the market returns (σ_M^2):

$$\beta_i = \frac{\sigma_{iM}}{\sigma_M^2} \qquad [2]$$

The result of the multiple linear regression can be summarized as follow (Table 1).

On the regression model used we have developed several analyses as shown in the following sub-sections.

Table 1 – Analysis results

	Dependent variable: BETA2022
ROE2022	-0.001***
	(0.0004)
ROA2022	0.015***
	(0.004)
FA/TA	0.138
	(0.442)
Bvofdebt	0.012
	(0.012)
COGS/TotCosts	-0.916**
	(0.409)
ΔEarnings	-0.000*
	(0.000)
Constant	1.594***
	(0.391)
Observations	95
R2	0.307
Adjusted R2	0.260
Residual Std. Error	0.892 (df = 88)
F Statistic	6.501*** (df = 6; 88)
P-value	1.052e-05
Note:	*p<0.1; **p<0.05; ***p<0.01

3.1 – *Multicollinearity analysis*

Multicollinearity occurs when two or more independent variables in the regression model are highly correlated with each other. To assess the presence of multicollinearity, we used the Variance Inflation Factor (VIF) tool (O'Brien, 2007). Therefore, if:

- VIF = 1: there is no correlation between one independent variable and the other independent variables;
- 1 < VIF < 5: there is a moderate level of correlation that it is commonly considered acceptable;
- VIF > 5: there is a high level of correlation, and it requires attention;
- VIF > 10: there is a very high level of correlation, and it implies the presence of a severe multicollinearity that compromise the results of the regression analysis.

In our analysis, the VIF values are the follows (Table 2):

Table 2. – Results of multicollinearity analysis

Variable	VIF
ROE2022	1.820188
ROA2022	1.763894
FA/TA	1.124744
Bvofdebt	1.027542
COGS/TotCosts	1.214942
ΔEarnings	1.385235
Mean	1.389425

The analysis shows that all the provided VIF values are less than 5, indicating that there are no significant issues of multicollinearity among the independent variables in our regression model. Therefore, it can be concluded that multicollinearity is unlikely to substantially influence the regression coefficient estimates in this model.

3.2 – *Autocorrelation analysis*

Autocorrelation occurs when the residuals of the regression model are not independent of each other. Following the models proposed by Granger (1969), Green (2003), and Hamilton (2020), to assess the presence of autocorrelation, we utilized the Durbin-Watson test where:

$$d = \frac{\sum_{t=2}^{T}(e_t - e_{t-1})^2}{\sum_{t=1}^{T} e_t^2} \qquad [3]$$

The Durbin-Watson statistic value always falls between 0 and 4 where:

- a value close to 2 indicates the absence of autocorrelation;
- a value below 1.5 or above 2.5 suggest the presence of strong positive or negative autocorrelation, respectively.

The associated p-value of the test indicates the probability of obtaining a Durbin-Watson statistic value as extreme as the observed one, assuming no autocorrelation in the data. A low p-value suggests sufficient evidence to reject the null hypothesis of no autocorrelation, while a high p-value suggests insufficient evidence to reject this hypothesis.

In our case, the Durbin-Watson statistic assumes a value of 1.9551 (see Table 3) and the associated p-value is equal to 0.3393 (greater than 0.05).

The absence of autocorrelation can also be observed in the autocorrelation plot where:

- on the x-axis, it is indicated the delays (lags) where a lag of 1 means we are comparing each point with the immediately succeeding point in the time series, a lag of 2 with the second succeeding point, and so forth;

- on the y-axis, it is indicated the autocorrelation values where the value shifts in the range between -1 and 1 (a value close to 1 indicates a strong positive correlation; a value close to -1 indicates a strong negative correlation; a value close to 0 indicates weak or no autocorrelation).
- significance lines: the dashed horizontal lines indicate the significance limits. If a bar falls outside these lines, the autocorrelation is considered statistically significant.

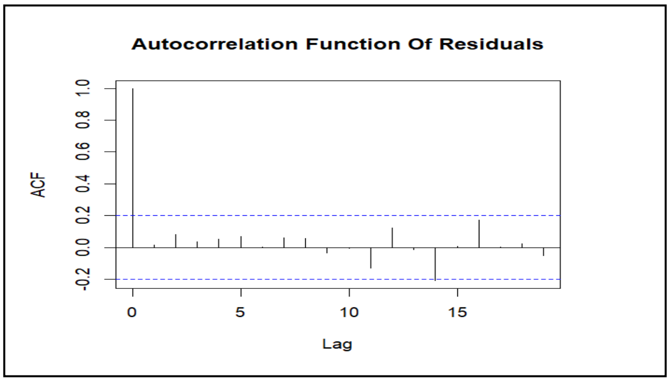
In our case, all the bars in the autocorrelation plot fall within the significance lines (see Table 3). Therefore, we have no significant correlations in the data beyond chance. In other words, there is no notable autocorrelation in the analyzed time series.

Table 3 – Durbin-Watson test and autocorrelation plot. ***Results of autocorrelation analysis***

Durbin-Watson test
=========================

Statistic:	1.95509159119493
P-value:	0.3393235718285
Method:	Durbin-Watson test
Data name:	mod

=========================



3.3 – *Heteroskedasticity analysis*

Heteroskedasticity occurs when the variance of errors is not constant (Wooldridge, 2010). To assess the presence of heteroskedasticity, we will use the following tests:

a) Breusch-Pagan Test (Breusch-Pagan, 1979): to assess whether the variance of errors is a function of the independent variables. A low p-value (i.e., less than 0.05) suggests the presence of heteroskedasticity;
b) White Test (White, 1980): a test that considers not only linearity but also interactions among the dependent variables;
c) Plot Residuals vs Fitted Values;
d) Plot Squared Residuals vs Fitted Values: to highlight variations in the dispersion of residuals.

Regarding the Breusch-Pagan test, it yields the following values:

- BP = 16.073;
- DF = 6;
- P-value = 0.01337.

Therefore, the results of the Breusch-Pagan test suggest that there is significant heteroskedasticity in the residuals of the regression model. It is further confirmed by the White test, which yields the following values:

- Chi-square = 26.5236;

- DF = 1;
- P-Value = 0.00000026034.

Results show a presence of heteroskedasticity in the residuals of the regression model. This suggests that the error variance is not constant.

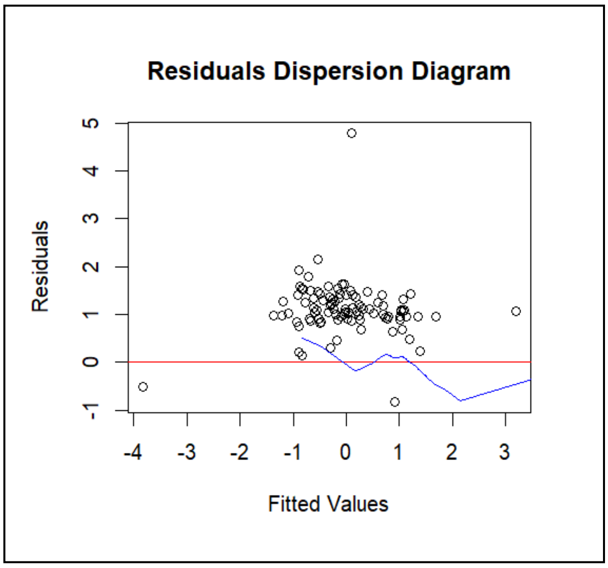
The presence of heteroskedasticity is also confirmed in the Residuals vs Fitted Values plot. In our case, the plot shows a concentration of residuals between -1 and 2, indicating a relatively small range, which is good. Additionally, it also shows the presence of a dashed blue line with both increasing and decreasing dashes, suggesting the likely presence of complex structure in the data that the linear model fails to capture accurately. Finally, the Squared Residuals vs Fitted Values plot shows the presence of squared residuals correctly ranging between 0 and 5, with a blue line trending downwards, indicating that the variance of residuals decreases as predicted values increase, suggesting the presence of heteroskedasticity.

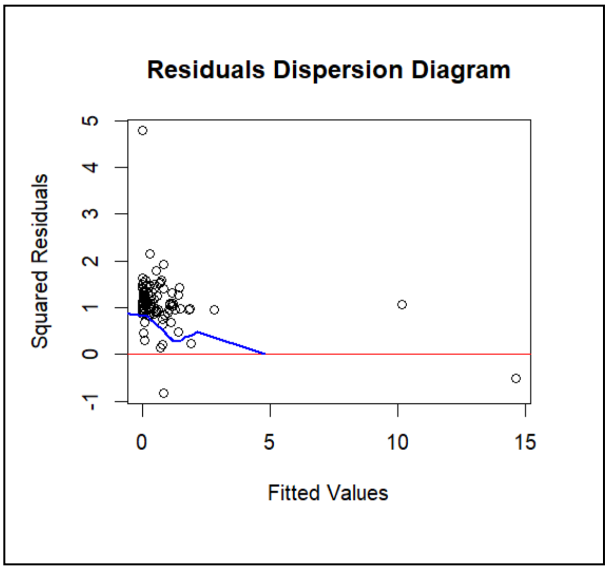
The results are summarized in Table 4:

Table 4 – Results of heteroskedasticity analysis

```
Breusch-Pagan test
==============================
Statistic BP:        16.073
DF:                  6
P-value :            0.0137
Method: Breusch-Pagan test
Data name: mod
==============================
```

```
Non-constant Variance Score Test
==============================
Chisquare:           26.5236
DF:                  1
P-value:             2.6034e-07
Method: White test
Data name: mod
==============================
```





In the presence of these values indicating heteroskedasticity, to address this issue and ensure reliable coefficient estimates, we applied the Huber-White robust covariance matrix correction (Huber, 1967; White, 1980). The estimated coefficients remained unchanged in both results, while the robust correction yielded larger standard errors compared to those obtained with the linear regression model, reflecting the heteroskedasticity present in the data and providing more reliable estimates for significance tests (see Table 5).

Table 5 – Huber-White Test

Regression with Robust Standard Errors

	Dependent variable: BETA2022
ROE2022	-0.001***
	(0.001)
ROA2022	0.015**
	(0.007)
FA/TA	0.138
	(0.456)
Bvofdebt	0.012***
	(0.003)
COGS/TotCosts	-0.916**
	(0.460)
ΔEarnings	-0.000**
	(0.000)
Constant	1.594***
	(0.390)
Observations	95
R2	0.307
Adjusted R2	0.260
Residual Std. Error	0.892 (df = 88)
F Statistic	6.501*** (df = 6; 88)
P-value	1.052e-05
Note:	*p<0.1; **p<0.05; ***p<0.01

This correction has thus allowed for obtaining robust standard errors, thereby improving the validity of statistical inferences on the model coefficients. The results suggest that inferences based on the corrected standard errors are more reliable compared to those based on uncorrected standard errors (Huber, 1967; White, 1980).

The results of this multiple linear regression analysis provide several practical and theoretical insights:

– First, the significant independent variables identified should be the focus of further studies and interventions, as they are the ones that most influence the dependent variable;

– Second, the absence of multicollinearity and autocorrelation in the residuals confirms the robustness of the model, making the coefficient estimates reliable.

However, the presence of heteroscedasticity requires a cautious approach to interpretations. The use of the robust Huber-White model represents an adequate solution for obtaining more accurate estimates, but it may be beneficial to further explore the causes of heteroscedasticity to further improve the model and ensure that the inferences are statistically valid.

4 – Conclusions

The regression model used show the main relevance of the variables used. Although the significance of the model is moderate ($R^2 = 0.31$ and adjusted $R^2 = 0.26$), this result is significantly higher than similar models used previously. For example, Damodaran (2012) obtained an R^2 of 0.093, showing low explanatory power in his model based on variables such as ROE and the ratio of fixed assets to total assets. Beaver, Kettler, and Scholes (1970) found a significant relationship between beta and some of the seven key variables they examined, such as leverage and asset growth, but the results were not completely consistent for all the variables analyzed. Compared with these studies, the main difference in our model lies in the use of variables such as the COGS/total cost ratio and the change in earnings (Δ Earnings), which showed a significant impact and improved the explanatory power of the model.

Therefore, the multiple linear regression model shows a moderate but significant explanatory power, with a good portion of the variability explained by the independent variables. The significance of the model is confirmed by the F-statistic and very low p-values. The absence of multicollinearity and autocorrelation in the residuals are positive elements that strengthen the model's validity. However, the presence of heteroscedasticity required the adoption of the robust Huber-White model to ensure reliable estimates.

Therefore, the model can be used to estimate the fundamental beta of the private firm on the basis of its variables.

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